Math 10B with Professor Stankova
Quiz 13; Tuesday, 5/7/2019
Section #203; Time: 11 AM
GSI name: Roy Zhao **Name:** ____________________

Circle True or False or leave blank. (1 point for correct answer, -1 for incorrect answer, 0 if left blank)

1. True **FALSE** It is possible to have a geometric random variable X have expected value 5 and variance 5 as well.

   Solution: If $E[X] = 5$ then $(1-p)/p = 5$ so $p = 1/6$ and $Var(X) = (1-p)/p^2 = 30$.

2. True **FALSE** The number of ways to place b balls into u boxes with $b > u$ is 0 if the boxes are indistinguishable and we want it to be injective but sometimes more than 0 if the boxes are distinguishable (still injective).

   Solution: Both are 0.

Show your work and justify your answers. Please circle or box your final answer.

3. (10 points) I have a bag with 5 coins and 4 are fair while one has both sides tails.

   (a) (5 points) Suppose you randomly reach in the bag and grab a coin and flip it. Suppose that you flip tails. What is the probability you have a fair coin?

   Solution: Using Bayes Theorem, it is

$$\begin{aligned} P(fair|T) &= \frac{P(T|fair)P(fair)}{P(T|fair)P(fair) + P(T|\overline{fair})P(\overline{fair})} \\ &= \frac{\frac{1}{2}\cdot\frac{4}{5}}{\frac{1}{2}\frac{4}{5} + 1\cdot\frac{1}{5}} \\ &= \frac{\frac{2}{5}}{\frac{2}{5}+\frac{1}{5}} = \frac{2}{3}. \end{aligned}$$

   (b) (5 points) Now suppose you flip it again and get tails again (total of two tails in a row). What is the probability it is fair now?

Solution: Using Bayes Theorem, it is

$$\begin{aligned}
P(fair|TT) &= \frac{P(TT|fair)P(fair)}{P(TT|fair)P(fair) + P(TT|\overline{fair})P(\overline{fair})} \\
&= \frac{\frac{1}{4} \cdot \frac{4}{5}}{\frac{1}{4}\frac{4}{5} + 1 \cdot \frac{1}{5}} \\
&= \frac{\frac{1}{5}}{\frac{1}{5} + \frac{1}{5}} = \frac{1}{2}.
\end{aligned}$$